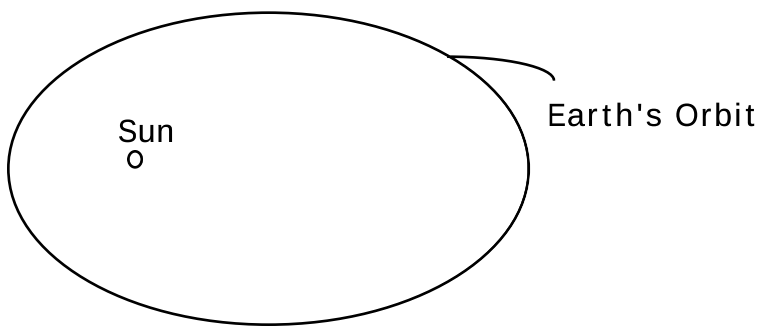
## Appendix A: Deriving the Equation for the Mass of Jupiter:

While this unit primarily uses the work of Isaac Newton, he was not working in a vacuum. Scientific theories and laws are built upon the work of many different people working from the beginning of time to understand the world around us. Determining the mass of Jupiter is no exception. The scientists who have received most of the credit for the rules about orbiting bodies are Tycho Brahe (born in 1546 to a Danish noble family), Galileo Galilei (born in 1564 in Pisa, Italy), Johannes Kepler (born on December 27, 1571 in Weil, Germany) and Isaac Newton (born in 1642 in England).

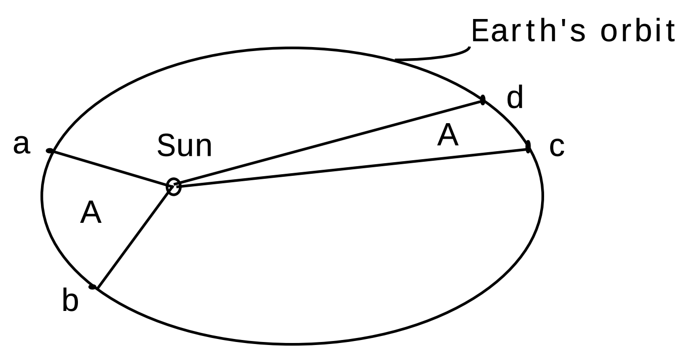
During the time from 1609 to 1619, Kepler announced three empirical laws describing the motion of planets in our solar system. (Empirical laws are laws derived from observation and experiment as opposed to theory which can come from thought alone.)

Kepler's Laws :

1. The Law of Ellipses : The orbit of each planet is an ellipse with the Sun at one focus.




2. The Law of Equal Areas : The line from the Sun to a planet sweeps out equal areas in equal intervals of time.




In the figure above the time interval from a to b is the same as the time interval from c to d for Earth as it orbits Sun. The areas swept out are equal to A in each case.

3. The Harmonic Law : The square of the sidereal period of each planet is proportional to the cube of the semi-major axis (mean radius) of its orbit.

This may be written algebraically as : $T^2 = k\,d^3$ , where T is the planet's period of revolution around the Sun, d is its average distance from the Sun, and k is the proportionality constant, which needs to be determined for each orbital system of objects. Our solar system of planets orbiting the sun has its own value of k and Jupiter with its moons has a different value of k.

Kepler had shown by 1621 that the four moons of Jupiter discovered by Galileo obeyed the relationship in the Harmonic Law (law #3).

**Deriving the Equation**

Given the first three equations, which are in the unit, here is the completion of the derivation of the equation for the mass of Jupiter.

When Newton realized that the gravitational force provides the centripetal force that makes celestial objects orbit each other, he could then equate the gravitational force law (3) with the centripetal force expression (1) to get the following:

$$(4) \qquad \frac{G\ M_J\ m_m}{d^2} = \frac{m_m\ v^2}{d}$$

This equation reduces to :

$$(5) \qquad \frac{G\ M_J}{d} = v^2$$

We now have the mass of Jupiter in our equation, so we are getting somewhere. If we could determine d and v, we would be able to get a value for the mass of Jupiter. Taking the relationship for v in equation (2) and substituting it into equation (5), we get:

$$(6) \qquad \frac{G\ M_J}{d} = \frac{4\,\pi^2\ d^2}{T^2}$$

We can now solve this for the mass of Jupiter to get :

$$(7) \qquad M_J = \frac{4\,\pi^2\ d^3}{G\ T^2}$$

The information we need to extract from our images to get the mass of Jupiter is the radius of a particular moon's orbit, d, and the orbital period of the moon, T.