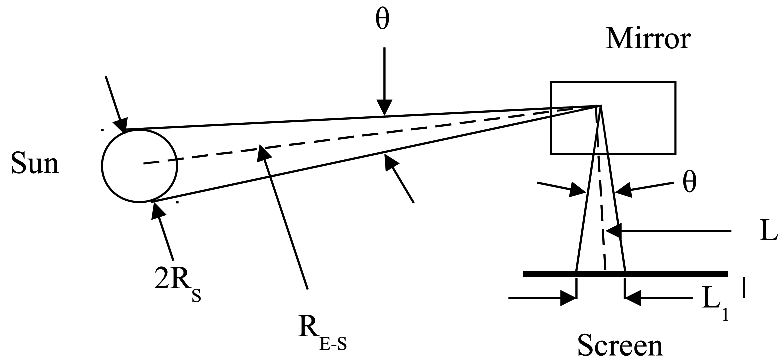
Teacher's Notes for Solar Constant Lab

These notes explain the angular diameter of the sun in more detail and also derives Eq. (2) for the solar constant . The figure below is not to scale, but shows the relationship between the angular size of the sun and the angular size of the image on the screen. The angles are both θ. The diameter of the sun is $2R_S$ (twice the radius) and the distance from the Sun to the Earth is $R_{E\text{-}S}$.



Using trigonometry,

$$\tan\frac{\theta}{2} = \frac{R_S}{R_{E-S}} \quad ;$$

for small angles,

$$\tan_{e\text{-}S}\frac{\theta}{2} \approx \frac{\theta}{2},$$

so

$$\frac{\theta}{2} \approx \frac{R_S}{R_{E-S}} \; .$$

By the same reasoning,

$$\frac{\theta}{2} \approx \frac{L_1}{2L},$$

or

$$\theta \approx \frac{L_1}{L},$$

where L_1 is the diameter of Sun's image on the screen (image diameter) and L is the distance from the mirror to the screen (mirror to image distance).

Now for the derivation of Eq. (2). The heat, Q, going into a system changes the energy of the system according to the equation

$$Q = mC_p\Delta T,$$

where m is the mass of the system, C_p is the specific heat of the system, and ΔT is the change in temperature of the system. In this experiment, the system is the water in the pan. The heat coming in is from the sun. Since power P is the amount of energy per unit time, the above equation can be changed into a power equation by dividing both sides by a change in time, Δt,

$$P = \frac{Q}{\Delta t} = mC_p \frac{\Delta T}{\Delta t}. \qquad (1)$$

The intensity, I, of light striking an area A is given by

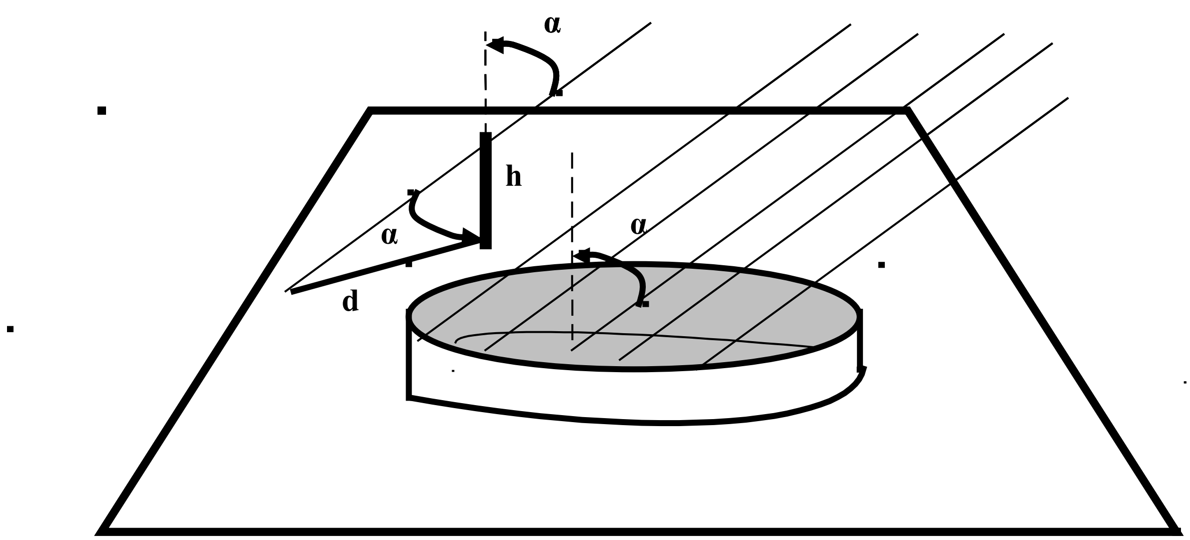
$$I = \frac{P}{A},$$

where P is the power of the light striking the area. The solar constant is the intensity of light striking the upper atmosphere, and, if there is no reflection by clouds or absorption in the atmosphere, the intensity of light striking the water in the pan is the same as the solar constant.

If the sunlight is striking the pan of water perpendicular to the surface of the pan, the power P of the light being absorbed is

$$P = SA,$$

where A is the area of the pan. However, it is very unlikely that the sunlight will strike the sunlight perpendicular to the surface of the pan. The situation is more likely to be the one shown in the figure below.



The sunlight is striking the pan at an angle α with respect to an imaginary line that is perpendicular to the surface of the pan. Thus the intensity of the light striking the pan is less than the intensity if the light was striking the pan perpendicular to the surface of the pan. (If your students have difficulty with this concept, there is a "Light Intensity Lab" that explores this idea.) The component of the light intensity that is perpendicular to the surface of the pan $S_\perp$ is

$$S_\perp = S\cos\alpha,$$

so $$P = S_{\perp}A = SA\cos\alpha.$$

Substituting into Eq. (1),

$$SA\cos\alpha = mC_P \frac{\Delta T}{\Delta t}. \tag{2}$$

Notice that the sun's rays and the stick make an angle α. Thus the cosine of α is h over hypotenuse of the right triangle formed by the stick and the shadow in the diagram. The hypotenuse is $\sqrt{(h^2)+(d^2)}$, so Eq. (2) becomes

$$\frac{SAh}{\sqrt{(h^2)+(d^2)}} = mC_P \frac{\Delta T}{\Delta t},$$

or

$$S = \frac{mC_P\sqrt{(h^2+(d^2))}}{Ah}\frac{\Delta T}{\Delta t}. \tag{3}$$

If $70^{\circ} \leq \alpha \leq 90^{\circ}$ corresponding to $d > \frac{h}{3}$, then $\sqrt{(h^2)+(d^2)} \approx d$, and

$$S = \frac{mC_P}{A}\frac{d}{h}\frac{\Delta T}{\Delta t} \tag{4}$$

which is Eq. (2) in the "Solar Constant Lab". If $d \leq \frac{h}{3}$, then Eq. (3) above should be used.