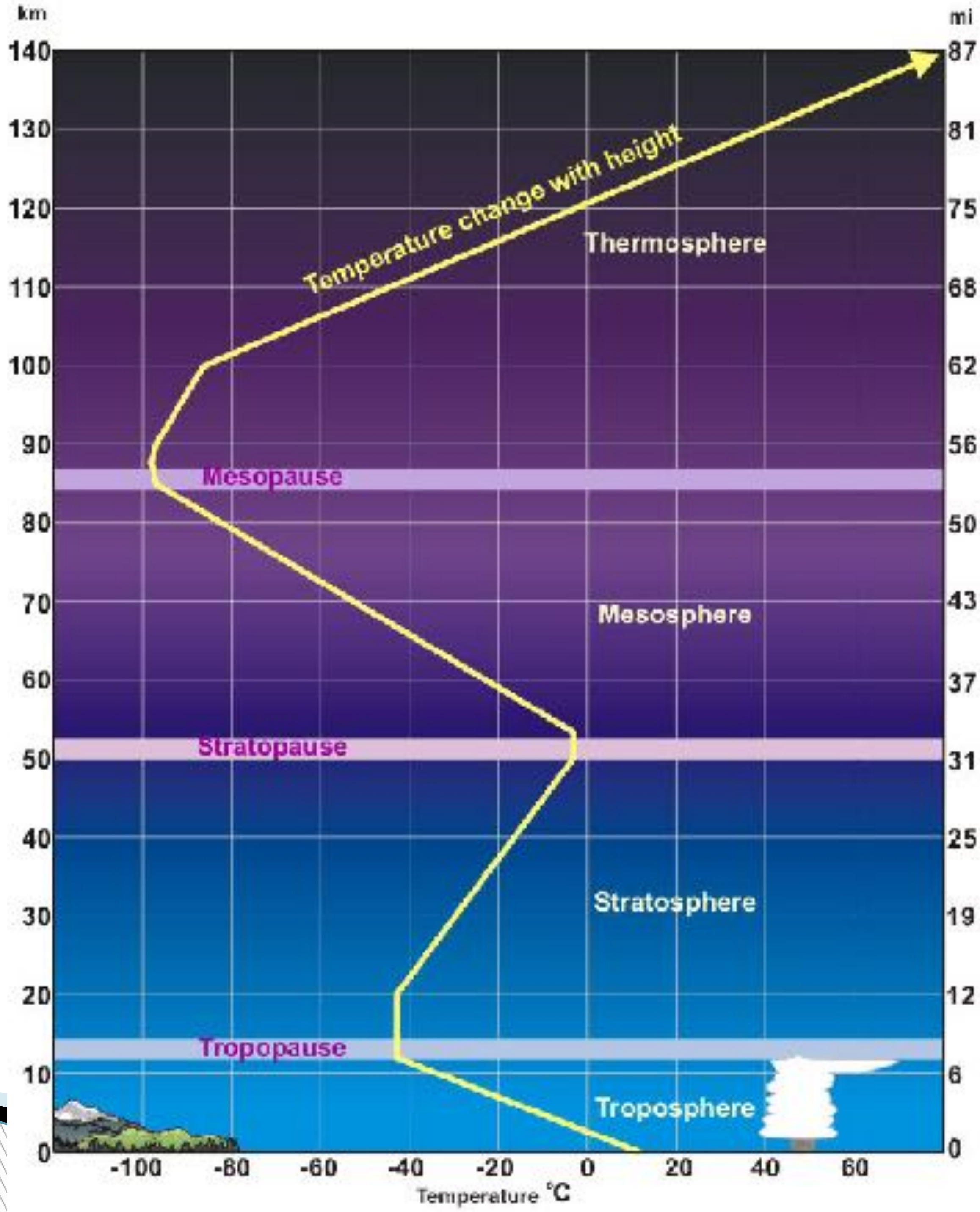
Equilibrium Temperature with Greenhouse Gases



km
mi
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
87
81
75
68
62
56
50
43
37
31
25
19
12
6
0
Temperature change with height
Thermosphere
Mesopause
Mesosphere
Stratopause
Stratosphere
Tropopause
Troposphere
-100
-80
-60
-40
-20
0
20
40
60
Temperature °C

$$\frac{\Delta T}{\Delta z} = -\Gamma$$

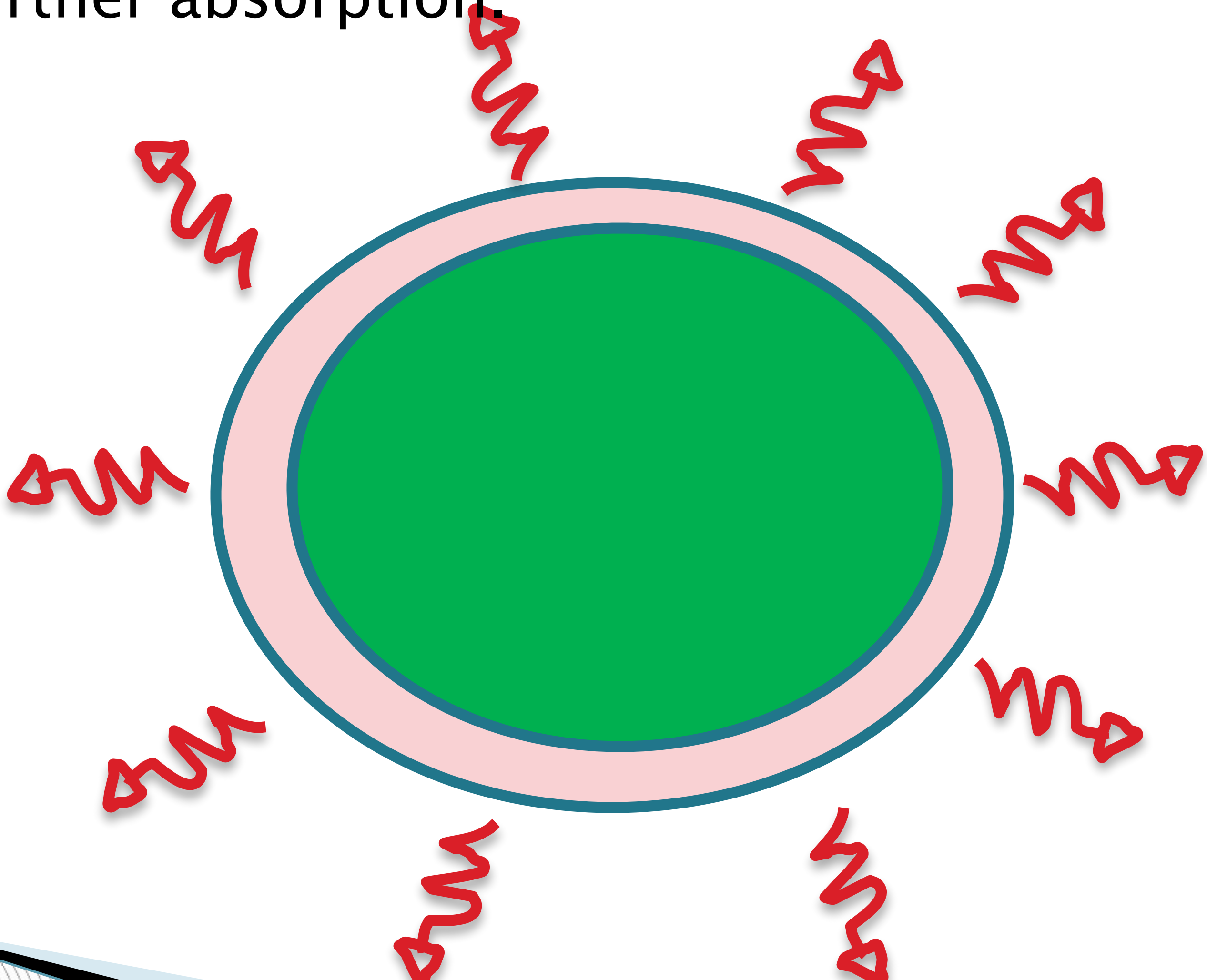
Equation for temperature change with altitude in the troposphere

Γ is called the lapse rate for the troposphere.

$$\Gamma = 6.7\,\frac{^{o}C}{km}$$

Average lapse rate for dry air in the troposphere

At this altitude, 95% of IR escapes to space without further absorption.

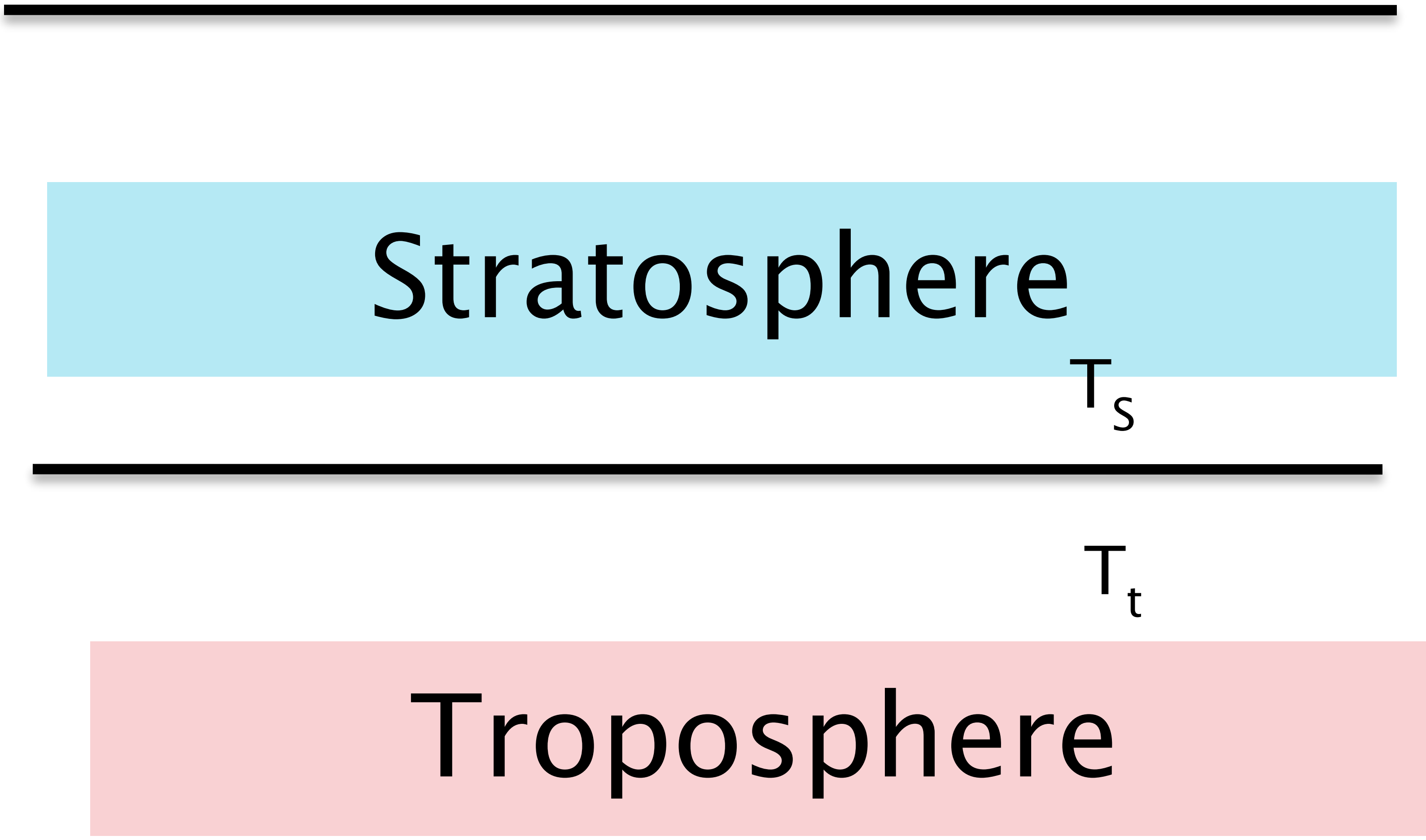


We want to find what the altitude is for 95% escape of IR,

And the surface temperature of the Earth with greenhouse gases.

We will use a two system model of the atmosphere. The two systems being the troposphere and the stratosphere.

We will assume thermal equilibrium in the stratosphere.


Stratosphere
T_s
T_t
Troposphere

We will assume the troposphere is a blackbody, so

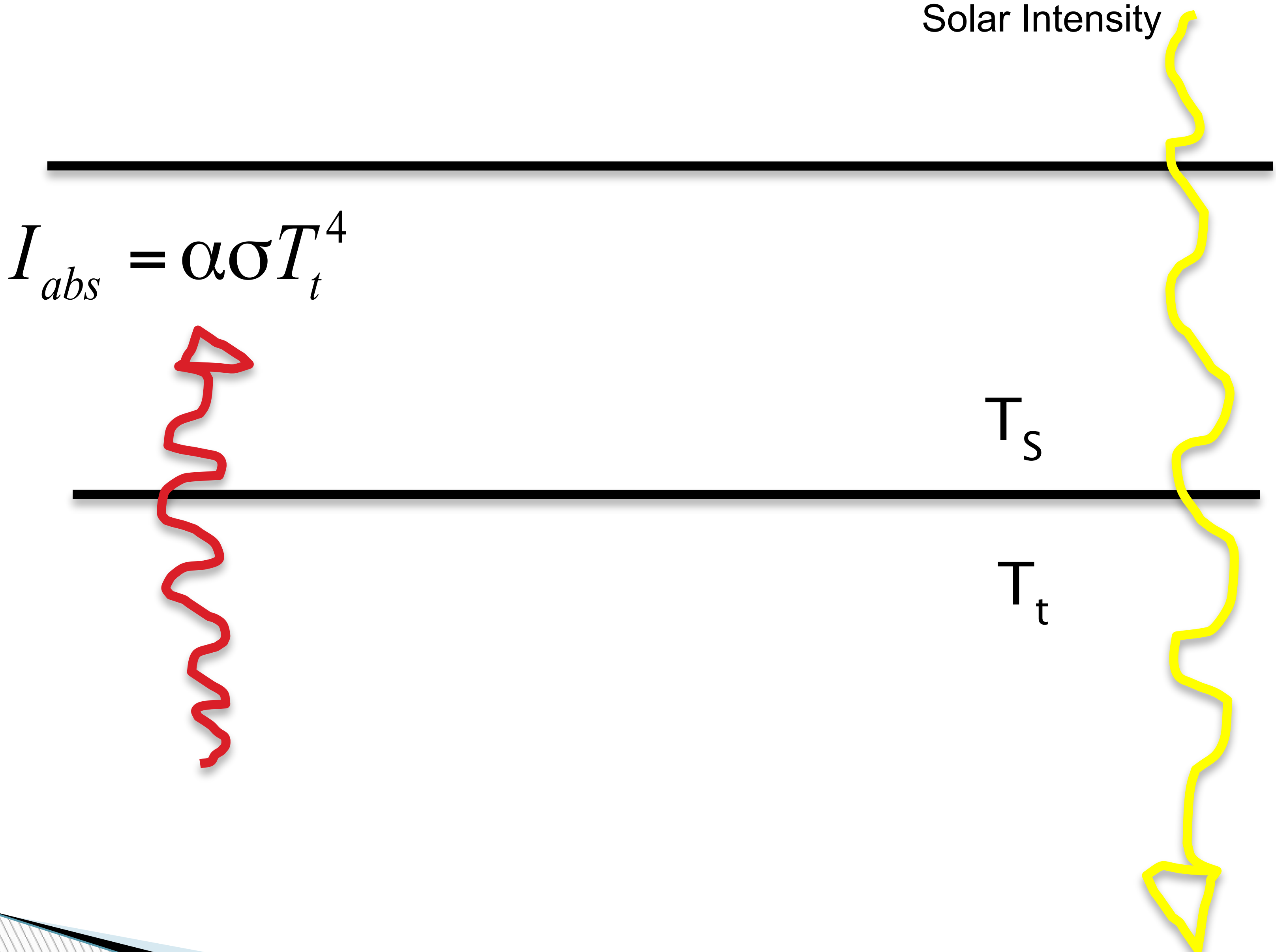
The radiation coming from the troposphere is

$$I_{rad} = \sigma T_t^4$$

The intensity entering the stratosphere is either absorbed or transmitted (passes all the way through).

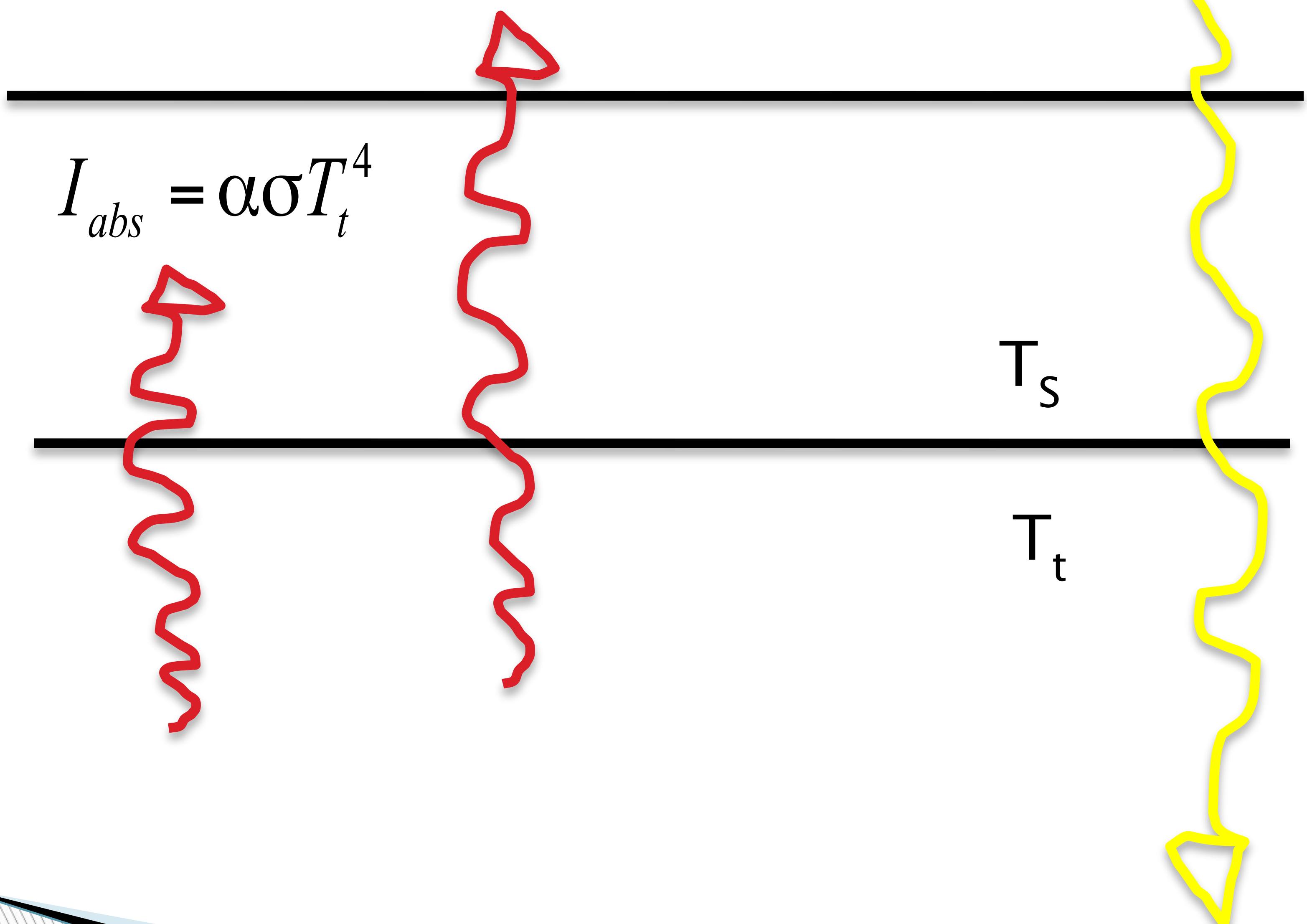
The intensity absorbed in the stratosphere is the intensity entering times the absorptivity of the stratosphere.

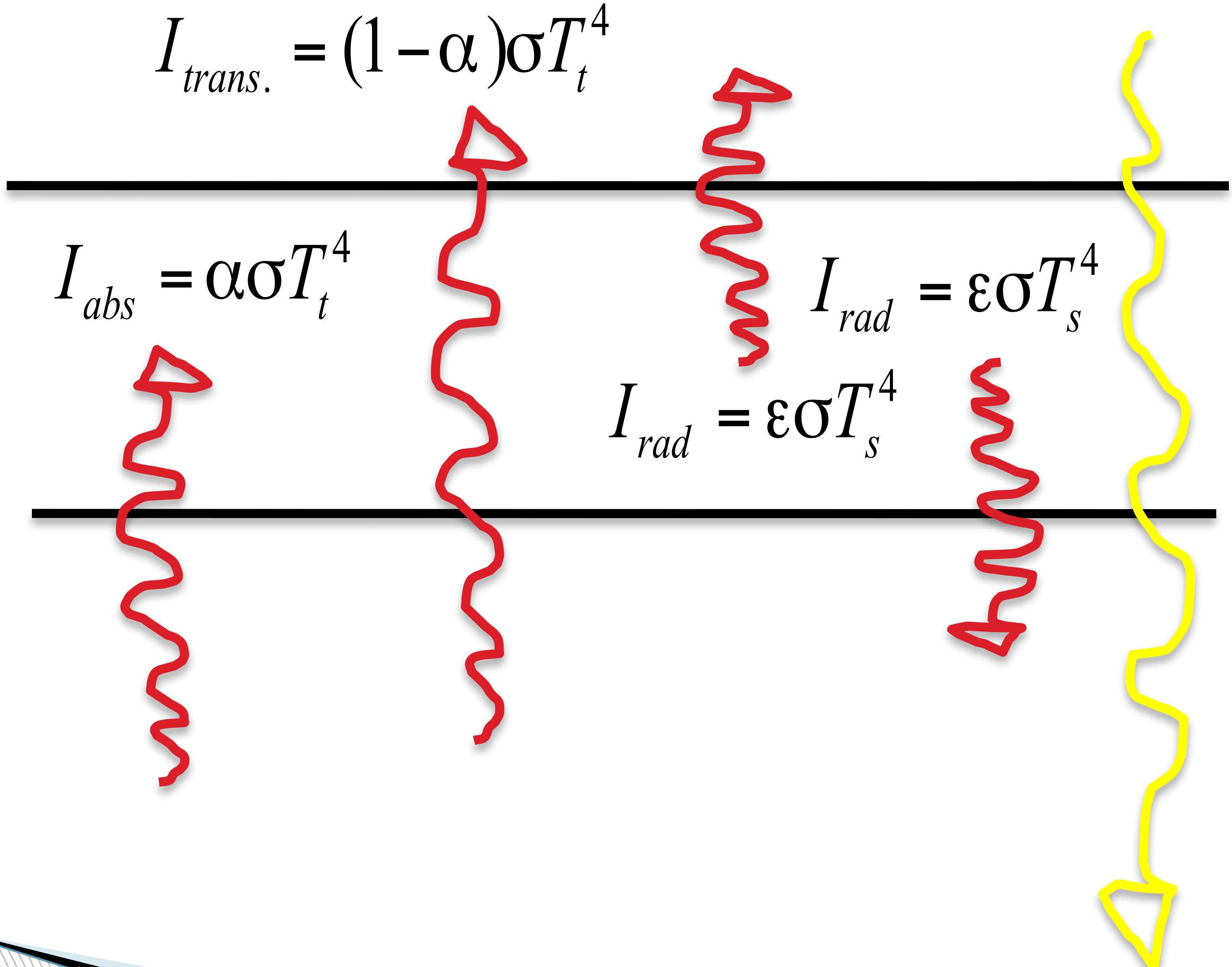
$$I_{abs} = \alpha \sigma T_t^4$$


Solar Intensity
$I_{abs} = \alpha\sigma T_t^4$
T_S
T_t

$$I_{trans.} = (1-\alpha)\sigma T_t^4$$

$$I_{abs} = \alpha\sigma T_t^4$$




$I_{trans.} = (1-\alpha)\sigma T_t^4$
$I_{abs} = \alpha\sigma T_t^4$
$I_{rad} = \varepsilon\sigma T_s^4$
$I_{rad} = \varepsilon\sigma T_s^4$

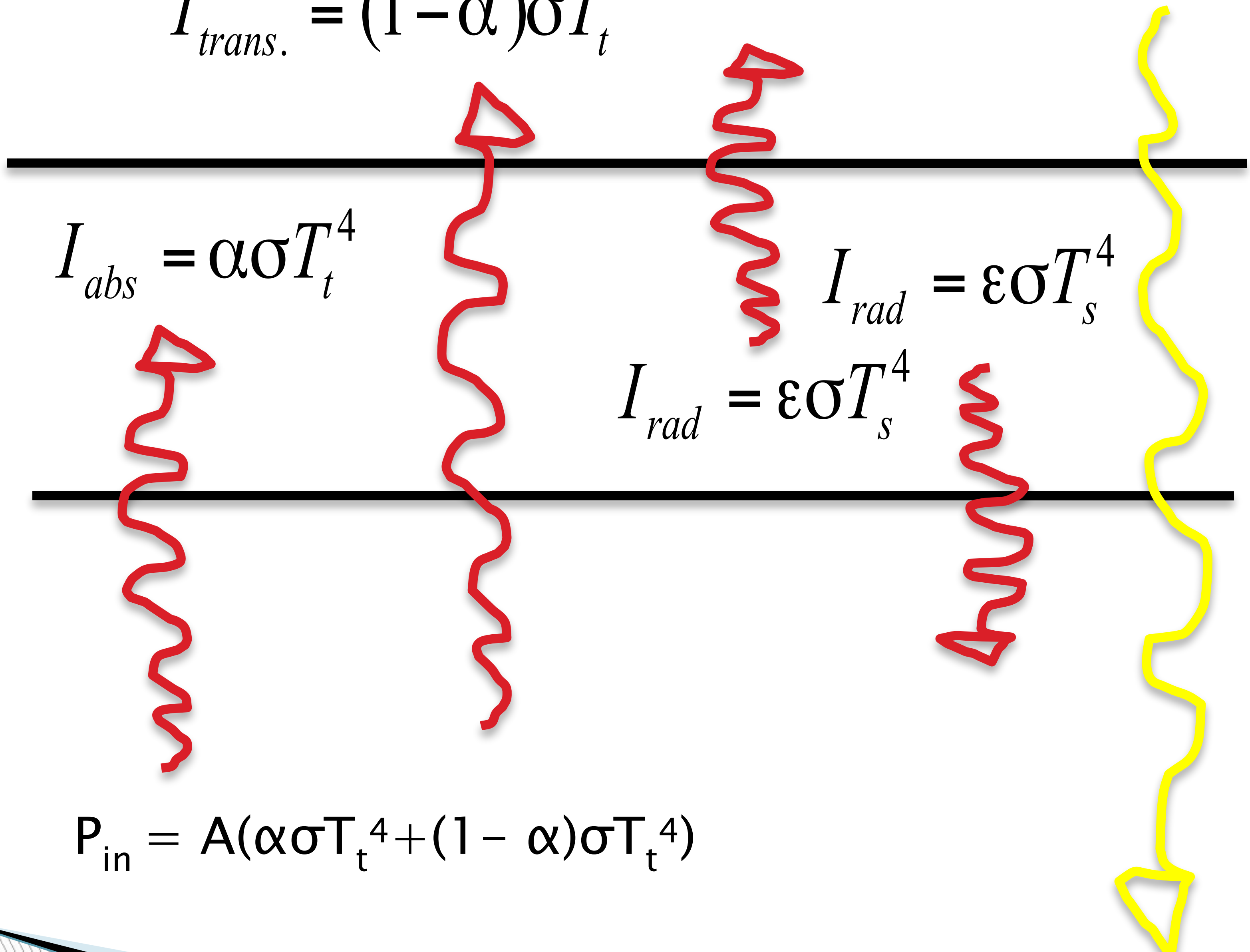
For thermal equilibrium,

power in = power out

$$P_{in}=P_{out}$$

For P_{in} we need to multiply all the intensities going in by area

For P_{out}, we need to multiply all the intensities going out by area.



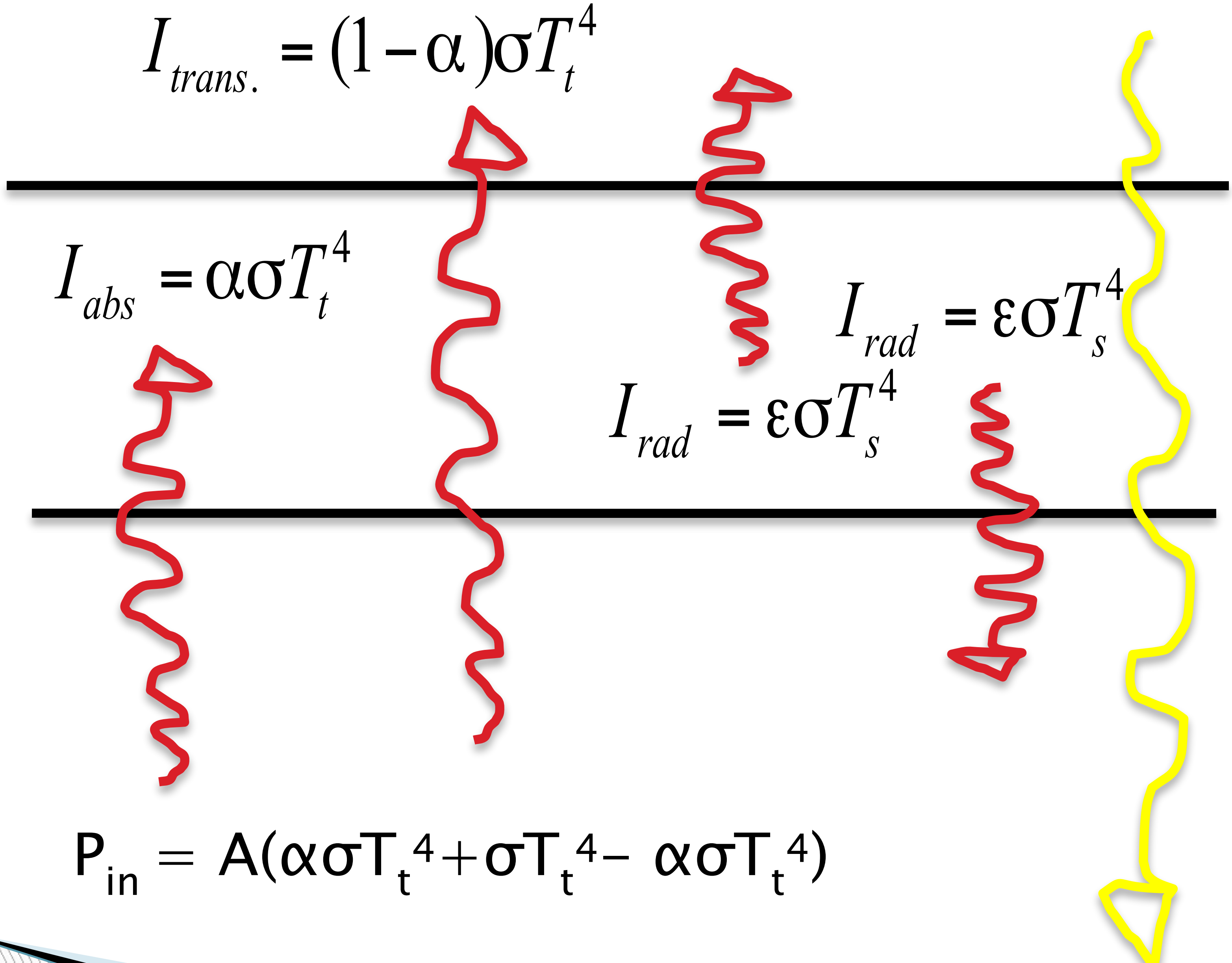
$$I_{trans.} = (1-\alpha)\sigma T_t^4$$

$$I_{abs} = \alpha\sigma T_t^4$$

$$I_{rad} = \varepsilon\sigma T_s^4$$

$$I_{rad} = \varepsilon\sigma T_s^4$$

$$P_{in} = A(\alpha\sigma T_t^4 + (1-\alpha)\sigma T_t^4)$$



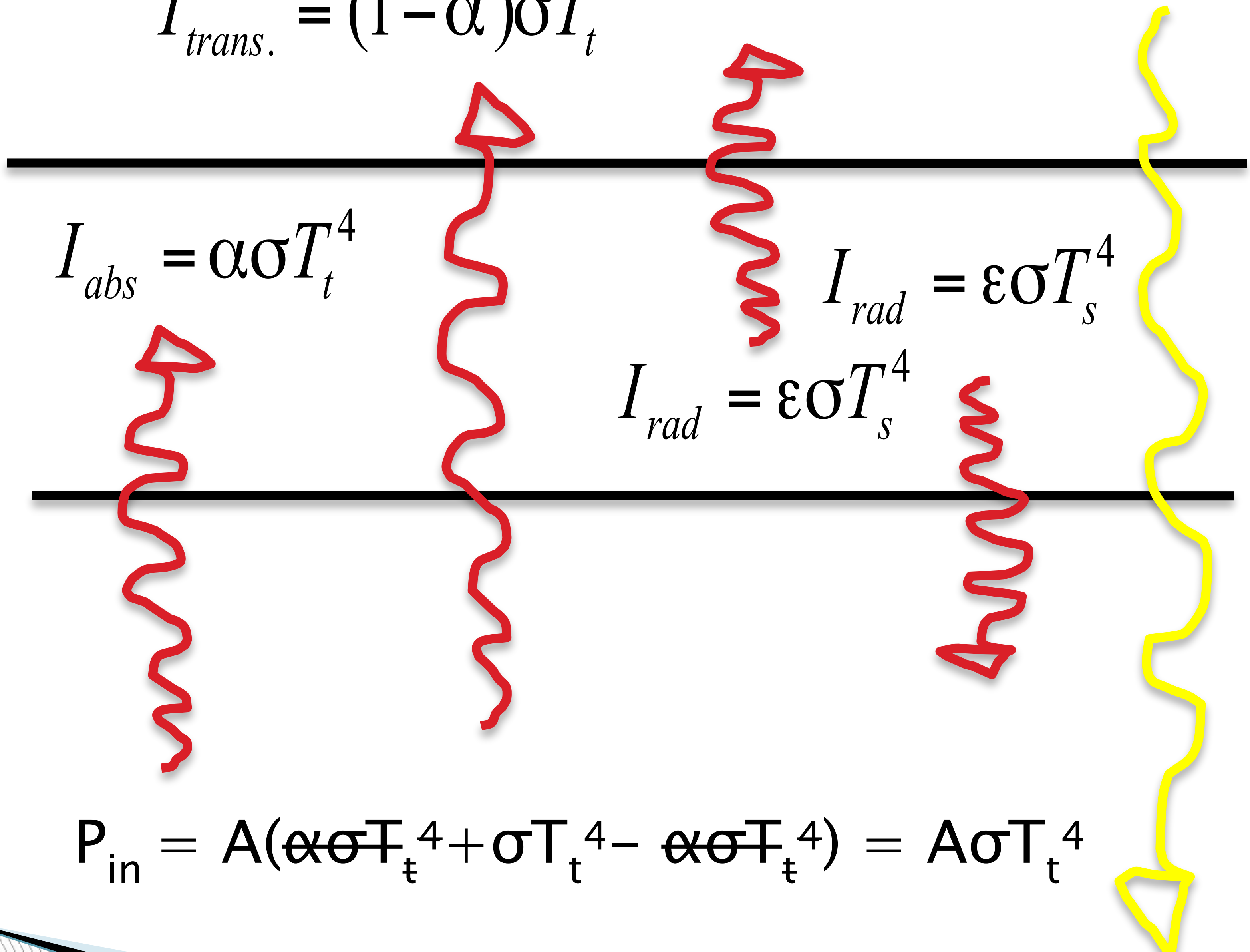
$$P_{in} = A(\alpha\sigma T_t^4 + \sigma T_t^4 - \alpha\sigma T_t^4)$$

$$I_{trans.} = (1-\alpha)\sigma T_t^4$$

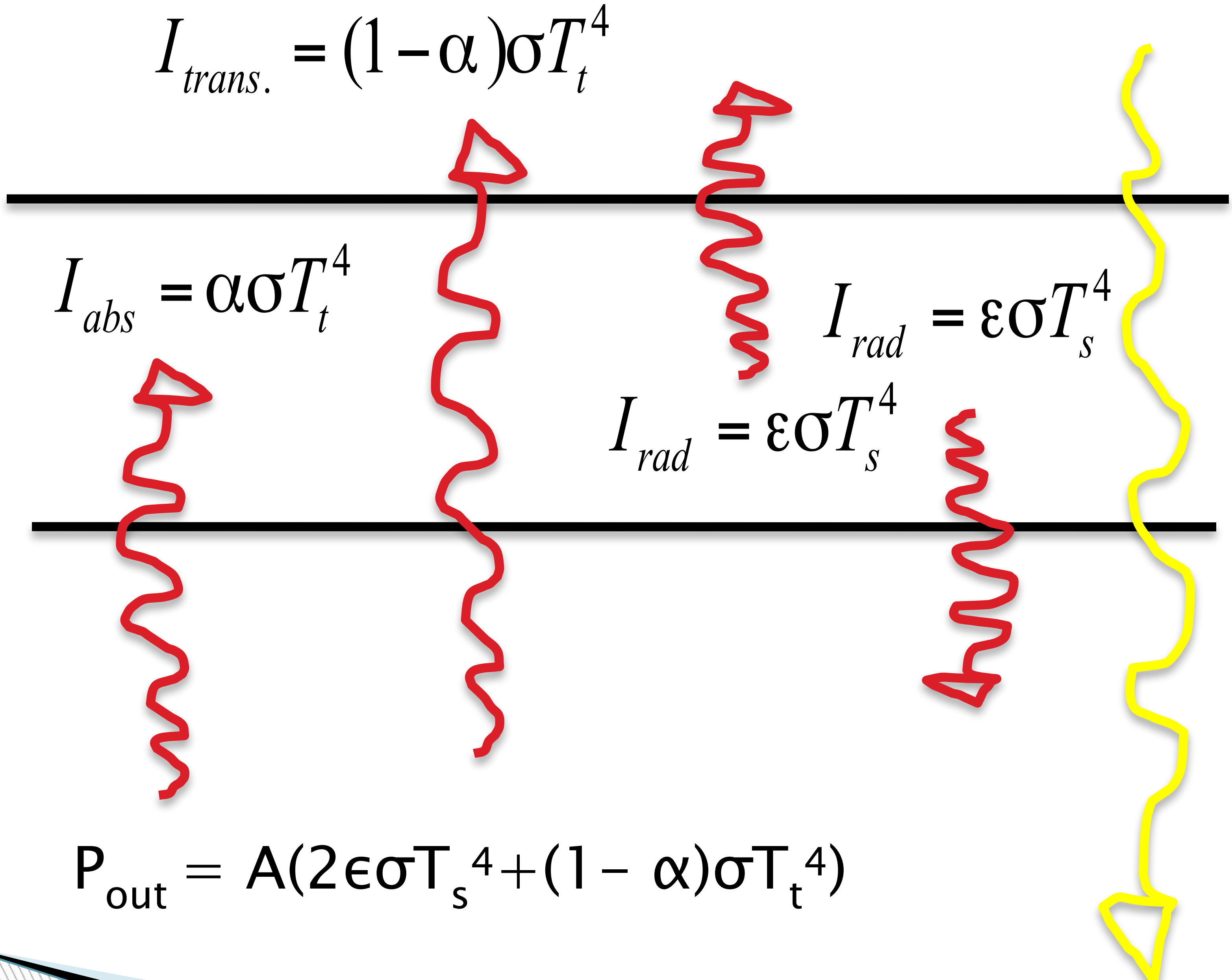
$$I_{abs} = \alpha\sigma T_t^4$$

$$I_{rad} = \varepsilon\sigma T_s^4$$

$$I_{rad} = \varepsilon\sigma T_s^4$$



$$P_{in} = A(\cancel{\alpha\sigma T_t^4} + \sigma T_t^4 - \cancel{\alpha\sigma T_t^4}) = A\sigma T_t^4$$



$$P_{out} = A(2\epsilon\sigma T_s{}^4 + (1-\ \alpha)\sigma T_t{}^4)$$

$$P_{in} = P_{out}$$

$$A\sigma T_t^4 = A(2\epsilon\sigma T_s^4 + (1 - \alpha)\sigma T_t^4)$$

$\epsilon = \alpha$, SO

$$\cancel{A}\sigma T_t^4 = \cancel{A}(2\alpha\sigma T_s^4 + (1 - \alpha)\sigma T_t^4)$$

$$\sigma T_t^4 = 2\alpha\sigma T_s^4 + \sigma T_t^4 - \alpha\sigma T_t^4$$

$$\cancel{\sigma T_t^4} = 2\alpha\sigma T_s^4 + \cancel{\sigma T_t^4} - \alpha\sigma T_t^4$$

$$0 = 2\cancel{\alpha\sigma}T_s^4 - \cancel{\alpha\sigma}T_t^4$$

$$0 = 2T_s^4 - T_t^4$$

$$0 = 2T_s^4 - T_t^4$$

$$+T_t^4 \qquad\qquad +T_t^4$$

$$T_t^4 = 2T_s^4$$

$$T_s = \frac{T_t}{(2)^{\frac{1}{4}}}$$

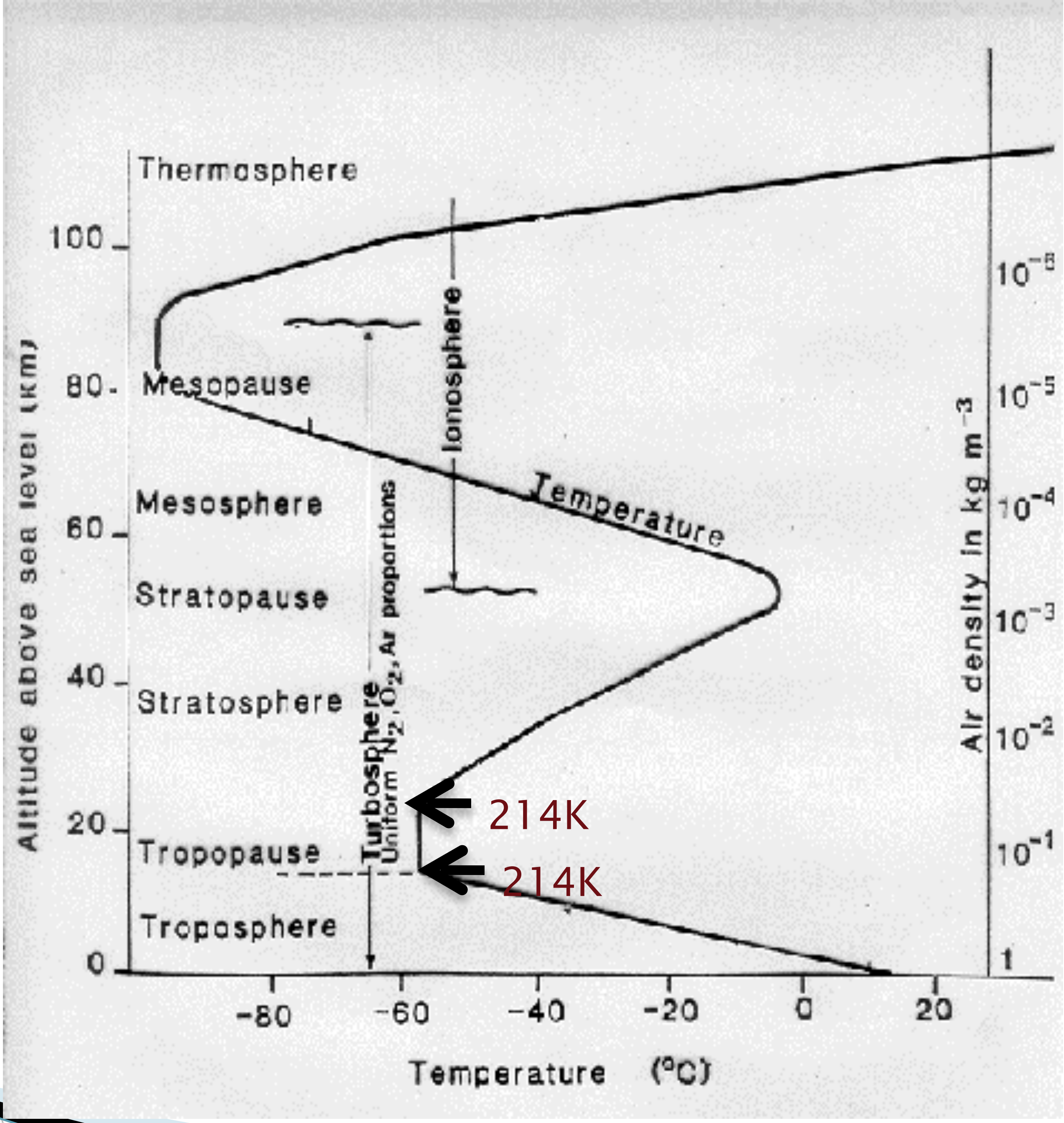
The temperature at which IR is escaping has to be 255 K because we have thermal equilibrium at this altitude with no absorption by greenhouse gases.

$$\text{Thus } T_t = 255 \text{ K}$$

$$T_s = \frac{255K}{(2)^{\frac{1}{4}}}$$

And the answer is

$$T_s = 214\ K$$



Thermosphere
Mesopause
Mesosphere
Stratopause
Stratosphere
Tropopause
Troposphere
Temperature
Ionosphere
Turbosphere
Uniform N_2, O_2, Ar proportions
214K
214K
Altitude above sea level (km)
Air density in kg m^{-3}
Temperature (°C)

Now let us find the altitude at which

$T_t = 255\ K$

$$\frac{\Delta T}{\Delta z} = -\Gamma$$

$$\frac{T_f - T_i}{z_f - z_i} = -6.7\,\frac{^{o}C}{km}$$

$$T_f - T_i = -6.7\,\frac{^oC}{km}\left(z_f - z_i\right)$$

The average top of the troposphere is $z_f = 11$ km, so

$$214K - T_i = -6.7\frac{^oC}{km}(11km - z_i)$$

$$214K - 255K = -73.7K + 6.7\frac{^{o}C}{km}z_i$$

$$-41K = -73.7K + 6.7\frac{^{o}C}{km}z_i$$

$+73.7K \quad +73.7K$

$$32.7K = 6.7\frac{K}{km}z_i$$

$$z_i = \frac{32.7K}{6.7\dfrac{K}{km}}$$

And the answer is

$$z_i = 4.9 \text{ km}$$

HOMEWORK:

1. Use $T_f = 214$ K and $z_f = 11$ km to find the surface temperature of the Earth with greenhouse gases.

If CO_2 doubles in the atmosphere, the top of the troposphere is predicted to rise to 11.6 km. What would the surface temperature of the Earth be then?